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Nucleon structure vs. theoretical models

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Abstract. An overview of nucleon structure as revealed by low-energy and intermediate-energy observables is presented. The role of seaquarks is identified as the complicating feature, which has prevented capture of the overall phenomenological features by quark models with simple interactions as well as by lattice calculations in the quenched approximation, however sophisticated. The large-N_C limit of QCD provides a simple and qualitatively satisfactory framework for the description the observables of the baryons and their lowest excitations.

PACS. 12.39.-x Phenomenological quark models – 14.20.Dh Protons and neutrons

1 Introduction

The successful description in terms of perturbative QCD of hadronic scattering observables at high energies and momentum transfer, has given rise to a widely held view, that nucleon and nuclear structure should also be describable by quarks that interact by the exchange of gluons. Thus "the strong force, which holds together protons and neutrons comes from the exchange of gluons" [1] and "because of gluons binding the atomic nucleus, matter is stable and doesn't crumble" [2]. What is still wanting, however, is a quantitatively convincing demonstration of reality of this view and of whether the gluons, as known from jet production in particle collisions at high energy, play any explicit role at all in the strong-coupling regime of nucleons and nuclei, or whether they are screened out.

The main reason why the early success of the constituent-quark model in describing the masses and magnetic moments of the ground-state baryons failed to generate a satisfactory overall understanding of baryon structure is the large role of seaquark configurations, which falls outside the framework of the 3-quark model. A simple method of estimating the role of these based on the empirical pionic decay widths is given in sect. 2. In sect. 3 the recent results obtained by numerical lattice calculations of the energies of the low-lying states in the baryon spectrum in the quenched approximation are described.

In sect. 3 recent quantitative attempts to describe the baryon spectrum as excitations of the QCD vacuum as described by the instanton liquid model of baryon observables in the large-N_C limit of QCD are reviewed.

In sect. 4 the large-N_C limit of QCD and its application to baryon structure is discussed.

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Table 1. Pion decay widths [3] of the lowest decuplet states (in MeV) and their ratios.

Decay	$\Gamma_{\rm exp}$	Ratio$_{\rm exp}$	$qqq_{\rm ratio}$
$\Delta \to N\pi$	120	12	9
$\Sigma^* \to \Sigma\pi$	39	3.9	4
$\Xi^* \to \Xi\pi$	10	1	1

2 Seaquarks and hidden degrees of freedom

The constituent-quark model posits that the flavor content of protons be uud and that of neutrons udd. The simplest seaquark configurations are admixtures of $u\bar{u}$ and $d\bar{d}$ to these basic 3-quark configurations. More exotic are the strangeness configurations $s\bar{s}$.

The simplest observable for the role of seaquarks is provided by the pion decay widths of the baryons with different strangeness. As pions do not couple to strange quarks (except through the small π^0-η mixing [4]), the pion decay widths of baryon resonances with similar symmetry should be proportional to the square of the number of light quarks. Thus, the ratio of the widths of the decuplet $\to$ octet $+ \pi$ decays in table 1 should be $9:4:1$. Empirically the decay widths of the Σ^* and the Ξ^* satisfy this scaling rule, their ratio being $4:1$, whereas the ratio factor for the Δ decay width is 12 rather than 9. This suggests that the strange hyperons are three quark states, with but insignificant seaquark configurations, while the Δ (and the nucleon) has an appreciable $qqq(q\bar{q})$ component.

If the amplitude of the $qqq(q\bar{q})$ component of the Δ is denoted y and that of the qqq component $1-y$, a linear analysis of the decay widths leads to the equation

$$3(1-y)+5y=\sqrt{12}, \quad (1)$$

from which $y = 0.23$. Thus, 23% of the Δ is a 5-quark rather than a 3-quark state. This result agrees perfectly with the well-known underestimate of 23% of the π-N-Δ coupling constant, for which the constituent-quark model gives the value $f_{\pi N\Delta} = 6\sqrt{2}/5 = 1.7$, whereas its value should be 2.1 in order to explain the empirical width.

Several experiments have found evidence for $\bar{u}$ and $\bar{d}$ quarks in the proton [5–8]. The most recent result of the E866 experiment may be summarized as

$$\int_0^1 [\bar{d}(x) - \bar{u}(x)] = 0.118 \pm 0.012. \quad (2)$$

This result reveals the $q\bar{q}$ sea to be flavor antisymmetric. The ratio $\bar{d}/\bar{u}$ is close to 1 at very small values of (Bjorken) x and grows to a peak value of ~ 1.7 at $x = 0.18$ and then drops below 1 at $x \simeq 0.3$. The x-dependence of this observable is not yet understood. The flavor asymmetry of the quark sea is however natural if one considers pionic fluctuations of the u and d quarks of the form $q \to q'\pi^0 \to q$, because in such fluctuations the number of $\bar{u}$ and $\bar{d}$ quarks in the loop fluctuations of the u and d quarks differ. Counting the antiquarks contained in the meson in the loop (under the assumption that it is a $q\bar{q}$ state) gives $\bar{d}/\bar{u} = 7/5 = 1.4$.

The role of seaquarks is even larger in the orbitally excited baryon resonances. A characteristic feature of the light flavor baryons and the strange hyperons is the ubiquity of low-lying positive-parity states. The lowest well-established orbitally excited states in the spectra of the nucleon, the Δ and the Σ are the positive-parity resonances $N(1440)$, $\Delta(1600)$ and the $\Sigma(1660)$. The $\Lambda(1600)$ also belongs to this group, although in the spectrum of the Λ-hyperon the two negative-parity flavor singlet states $\Lambda(1440)$ and $\Lambda(1520)$ lie below it.

In any monotonic confining well, the lowest orbitally excited resonances would have negative parity. The color hyperfine interaction, which arises from single-gluon exchange cannot shift the positive-parity states below the lowest negative-parity states, because the color-spin symmetry of these qqq states is the same, and hence the color-spin interaction cannot reverse the order of the spectrum [9].

Seaquark configurations are likely to be a main source of this puzzle. In table 2 the pion decay widths of the low-lying positive-parity states are compared. If these resonances were pure 3-quark states, the ratio of the pion decay widths of the non-strange positive-parity states to the strange ones should be 9 : 4. Empirically it is more like 17 : 4, which suggests that both the $N(1440)$ and the $\Delta(1600)$ contain substantial $qqq(q\bar{q})$ admixtures. A linear analysis as in eq. (1), but with $\sqrt{17}$ on the r.h.s. gives $y = 0.56$. The amplitude for the 5-quark configuration in these states is thus 56% or more than half. This result also explains why in the quark model, with P-wave coupling between pions and quarks, the N-$N(1400)$ transition coupling is only about a third of the value required to explain the empirical decay width for $N(1440) \to N\pi$ [10].

Table 2. Pion decay widths of the lowest positive-parity states (in MeV) [3] and their (approximate) ratios.

Decay	Γ_{exp}	$\text{Ratio}_{\text{exp}}$	qqq_{ratio}
$N(1440) \to N\pi, \Delta\pi$	350	17	9
$\Delta(1600) \to N\pi, \Delta\pi, N(1440)\pi$	350	17	9
$\Lambda(1600) \to \Sigma\pi$	~ 80	~ 4?	4
$\Sigma(1660) \to \Lambda\pi, \Sigma\pi$	~ 80	~ 4	4

In contrast to the low-lying positive-parity states, the lowest-lying negative-parity nucleon resonances are mainly qqq states. This may be inferred from the fact that ratios of the single-pion decay widths of the $N(1535)$ $1/2^-$ and $N(1520)$ $3/2^-$ states to the corresponding Λ excitations —the $\Lambda(1670)$ and the $\Lambda(1690)$— are about 13 : 4. If these states were pure qqq states the ratios would be 9 : 4, so again there is a significant ($\sim$) 30% , but far from dominant $qqq(q\bar{q})$ amplitude. Attempts have also been made to describe the $N(1535)$ and the $N(1520)$ resonances as meson-nucleon resonances without any qqq content [11].

3 Lattice calculation results

Recently, lattice calculations of the baryon spectrum in the quenched approximation, in which only 3 valence quark configurations are considered, have become available even for fairly small quark mass values. For the nucleon and its lowest negative-parity excitation ($N(1535)$) the calculated energy values, if linearly extrapolated to the physical quark mass values, appear to converge onto the empirical values [12,13]. This result suggests that the seaquark configurations are not much more significant in the $N(1535)$ state than in the nucleon. This conclusion agrees with the conclusion based on the ratios of the pionic decay widths above.

The published quenched lattice calculations do not, however, for the quark mass range hitherto accessible, show any convergence towards the positive-parity states, which empirically lie below the negative-parity states. A linear extrapolation of the calculated values in refs. [12, 13] leads to masses above 2 GeV, in gross disagreement with experiment. Given the argument above, that these states have larger amplitudes for $qqq(q\bar{q})$ than for qqq, this failure is perhaps not all that surprising, as the 5-quark configurations are left out by definition in the quenched approximation.

The validity of the quenched approximation has been tested by a comparison of the nucleon and Δ-resonance masses obtained in a dynamical calculation to the values given by the quenched approximation [14]. At small quark mass values the quenched approximation was found to overestimate the nucleon mass by about 10%, whereas the mass of the overestimate of the Δ mass was about twice that. This result suggests that the error of the quenched approximation grows with excitation energy, again in line with the increasing role of the seaquark configurations.

This may also be the reason for the lack of any signal for the low-lying flavor singlet $\Lambda(1405)$ in the quenched lattice calculation [13], as this state is likely to have a substantial $\bar{K}N$ component [15]. In the quenched approximation the calculated baryon spectrum is in fact qualitatively very similar to that, which obtains in qqq model with a gluon exchange hyperfine interaction [16].

A very recent lattice calculation of the low-lying baryon resonance spectrum, which employs Bayesian statistics to extract the lowest states shows that it may still be possible to obtain realistic spectra in the quenched approximation [17]. This calculation shows a sudden decline in the predicted masses of the positive-parity resonances, when the quark mass falls below values, that correspond to pion mass values of about 600 MeV. This calculation led to realistic energies for both the $N(1440)$, the $\Delta(1600)$ and the $\Sigma(1660)$ positive-parity resonances. Moreover the results strongly suggest that the $\Xi(1690)$ should have positive parity. Finally a satisfactorily low value for the energy of the $\Lambda(1405)$ was also obtained.

4 The role of instantons

Given the restriction that the lattice calculation results obtained in the quenched approximation have quantitative value for ground-state baryons, the predictions for those merit particular attention. A significant such result is the numerical demonstration that even in the quenched approximation a non-zero value for the quark condensate $\langle q\bar{q}\rangle$ obtains [18]. This result shows that the approximate chiral symmetry of QCD is spontaneously broken, and that the Gell-Mann-Oakes-Renner relation between the pion mass and the quark mass and quark condensate holds.

A recent lattice calculation of the dependence of the quark mass on the quark momentum shows how at ~ 2 GeV it begins to grow smoothly until it reaches the expected constituent value of ~ 300 MeV as the momentum decreases to 0 [19]. The simplest model, which includes the two key features of the spontaneous breaking of chiral symmetry and dynamical constituent mass, is the instanton liquid model of the QCD vacuum [20,21].

In the instanton liquid model the instantons induce a pointlike interaction between quarks. This provides the required splitting between the η and η' mesons, and has been suggested as a model for the hyperfine splittings of the baryon spectrum [22]. If the Bethe-Salpeter equation for a $q\bar{q}$ state is solved with this interaction, the isovector pseudoscalar (pion) state drops to very low mass, whereas the isovector vector state (ρ) meson mass stays around twice the constituent mass, as desired.

A characteristic feature of the instanton-induced interaction, is that it only acts in states which are antisymmetric in the quark flavor. Because of this it is inactive in the spectrum of the Δ, and cannot move the mass of the lowest positive-parity excitation $\Delta(1600)$ below that of the lowest negative-parity state $\Delta(1700)$ [23].

Recent calculations of the baryon spectrum with a relativistic quark model with linear confinement and the instanton-induced interaction confirm that the model cannot reproduce the empirical Δ spectrum [24]. The ordering of the low-lying positive- and negative-parity states in the nucleon spectrum also disagrees with the empirical ordering. Finally this model overestimates the energy of the flavor singlet $\Lambda(1405)$ by more than 100 MeV [24]. These problems may arise mainly because of the neglect of the 5-quark configurations rather than from a failure of the instanton liquid model itself. The spectrum obtained with this model is quite similar to that, which obtains with linear confinement and a (strong) single-gluon exchange model for the hyperfine interaction between quarks [25].

5 QCD in the large-N_C approximation

The large color limit of QCD provides a framework, which captures many of the key phenomenological features of baryon and nuclei. In this limit amplitudes that are described by diagrams with non-planar gluon diagrams are suppressed by at least one power of $1/N_C$ in comparison with planar diagrams in the S-matrix. For planar diagrams gluon lines may be represented by parallel $q\bar{q}$ lines, which admit a meson interpretation. Among the ramifications of this is the establishment of a framework for the meson exchange description of nuclear forces [26,27].

In the limit of large N_C, mesons and baryons are described as

$$M = \sum_{i=1}^{N_C} \bar{q}_i q_i, \quad B = \sum_{i_1,..,i_{N_C}} \epsilon_{i_1,...i_{N_C}} q_{i_1}...q_{i_{N_C}}. \tag{3}$$

The meson mass scales as $\sim N_C^0$, whereas the baryon mass scales as $\sim N_C \Lambda_{\rm QCD}$. The meson decay constants scale as $\sim \sqrt{N_C}$ and three-meson vertices as $\sim 1/\sqrt{N_C}$ [28]. As mesons consequently are stable in the large-N_C limit, this limit lacks utility for meson phenomenology.

The situation is different for baryons, however, as meson-baryon vertices scale with N_C, which in combination with the inverse meson decay constants in the couplings implies that meson-baryon couplings scale with $\sqrt{N_C}$ so that the meson-baryon couplings are strong in this limit.

There are several ways of realizing the large-N_C limit for baryons. One is the extension of the constituent-quark model to large N_C, and another is the bosonic method, in which the baryons are described as topologically stable soliton solutions to a chiral meson field theory.

The simplest version of the latter approach is the Skyrme model [29]. This model yields a correctly ordered spectrum with low-lying positive-parity states about 400 MeV above the nucleon and the Δ [30]. The lowest negative-parity states fall about 100 MeV above the positive-parity states, in agreement with experiment [31]. The Skyrme model describes the positive-parity baryon resonance as collective vibrations, in which seaquarks by definition play a major role. This is the most likely reason for the satisfactory spectroscopy of the model.

The original version of the Skyrme model is however too restrictive when it comes to the form factors of the nucleon. In that model the isoscalar and isovector form factors are related as [32]

$$G_M^S(q^2) = -\frac{2m_N}{\Omega}\frac{\partial G_E^S(q^2)}{\partial q^2},$$
$$G_E^V = \frac{1}{m_N \Omega}\left(\frac{3}{2}G_M^V(q^2) + q^2\frac{\partial G_M^V(q^2)}{\partial q^2}\right), \tag{4}$$

where Ω is the (constant) moment of inertia of the soliton. The first one of these relations implies that the isoscalar magnetic form factor falls faster with q^2 than the isoscalar electric form factor, whereas the second implies that both isovector form factors have the same rate of falloff. In view of the recent empirical indication that the electric form factor of the proton falls at a considerably faster rate with momentum transfer than the magnetic form factor [33], the relations (4) have to be restricted to very low values of momentum transfer. In order to describe the form factors at higher values of momentum transfer, more general versions of this approach would be called for [34,35].

The more direct approach to phenomenology based on the large-N_C limit is to employ quark operators [36]. Consider the following spin and isospin quark operators:

$$J^i = \sum_m q_m^\dagger \frac{\sigma^i}{2} q_m,$$
$$T^a = \sum_m q_m^\dagger \frac{\lambda^a}{2} q_m,$$
$$G^{ia} = \sum_m q_m^\dagger \frac{\sigma^i}{2}\frac{\lambda^a}{2} q_m, \tag{5}$$

where λ^a are flavor matrices. To order $1/N_C^2$ the following mass operators then apply for the baryon states in the ground-state band [28]:

$$M^1 = N_C + \frac{1}{N_C}\mathbf{J}^2,$$
$$M^8 = T^8 + \frac{1}{N_C}\{J^i, G^{i8}\} + \frac{1}{N_C^2}\{\mathbf{J}^2, T^8\},$$
$$M^{27} = \frac{1}{N_C}\{T^8, T^8\} + \frac{1}{N_C^2}\{T^8, \{J^i, G^{i8}\}\},$$
$$M^{64} = \frac{1}{N_C^2}\{T^8, \{T^8, T^8\}\}. \tag{6}$$

This lead to mass expressions, which may be algebraically combined to relations of equal order in $1/N_C$. The empirical baryon masses satisfy these relations very well, and moreover the degree of violation almost uncannily follows the powers of $1/N_C$ [36]. If expressed in percent, the ratios of the mass combinations that are of order $1/N_C$ to (the half of) the sums of the corresponding magnitudes are $\sim$ 18, 6 and 0.4, when ordered according to the corresponding power in $SU(3)$ flavor breaking, whereas at order $1/N_C^2$ the corresponding ratios are only 1, 0.2 and 0.1 —*i.e.* they are smaller by one order of magnitude [36].

The large-N_C operator analysis has also applied to the P-shell non-strange baryons [37] and strangeness-1 hyperons [38]. In the fits to the empirical masses only two of the linearly independent operators that appear to order $1/N_C^2$ only obtain large coefficients. These are equivalent to the scalar and the flavor-spin operators that were found to provide an overall description of the whole baryon spectrum in ref. [9], and which have an obvious meson exchange interpretation. An extension to the positive-parity resonances of this analysis should be instructive.

6 Discussion

While the substantial role of $u\bar{u}$ and $d\bar{d}$ configurations in the nucleon and Δ states is thus well established, searches for signatures in nucleon form factors of $s\bar{s}$ admixtures by means of parity-violating electron-nucleon scattering indicate that the strangeness contributions are very small, if not zero [39,40]. These findings are in line with what the constituent-quark model would suggest [41]. The only lattice calculations of the strangeness magnetic moments have been made in the quenched approximation, and have led to large negative values [42–44], which disagree with the extant empirical values.

The significant role of non-strange seaquark configurations in the nucleon is however well established. The large effect of the seaquark components on observables explains why the constituent-quark model with 3 valence quarks cannot provide a fully satisfactory description of nucleon and nucleon resonance structure.

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